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Final Exam , Math 530, Fall 2014

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QUESTION 1. Part 1: Questions to be answered outside class from i to iv.

(i) Show that there is no simple group of order 992. [hint: write $992 = 2^5.31$]

(ii) Suppose that $|G| = m.q_1^{n_1}.....q_k^{n_k}$, where m is an integer > 1, $q_1, ..., q_n$ are distinct prime numbers such that each $q_i > m$. Suppose that G has a subgroup , say H, of order $q_1^{n_1}.....q_k^{n_k}$. Prove that H is a normal subgroup of G.

(iii) Let G be an infinite simple group, and let H be a subgroup of G. Prove that the index of H in G is infinite, i.e., show that $[H : G] = \infty$.

(iv) Give me an example of an infinite group G that has a subgroup L such that $[L : G]$ is finite.

================= Part 2: IN CLASS QUESTIONS==

(v) Let G be an abelian group of order p^n, where p is prime and $n \geq 2$. Suppose that G has a unique subgroup of order p^k for some integer $1 \leq k < n$. Prove that G is cyclic.

(vi) Let G be an abelian group of order p^7, where p is prime. Suppose that G has a unique cyclic subgroup of order p^4. Find all non-isomorphic groups of order p^7 that satisfy this property. Briefly justify your answer

(vii) Let G be a group of order $2^2.3^k$ for some integer $k \geq 2$. Prove that G is not simple. [Hint: somehow you need to build a group homomorphism F from G into S_n for some n such that $Ker(F) \neq \{e\}$]

(viii) Prove that $L = Z_8 \times Z_{10}$ is group-isomorphic to $Z_2 \times Z_{40}$. How many elements of order 20 does L have?.

(ix) Given $|G| = p^n m$ for some prime p such that $n \geq 2$. Suppose that G has a unique subgroup, say H, of or order p. Prove that H is a normal subgroup of G. If G/H is cyclic, then prove that G is cyclic.

(x) Given G is a noncyclic group of order 17.11^2. Prove that G is abelian. Find all noncyclic non-isomorphic groups of order 17.11^2.

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